S.W. Drury

G. Schmidt

FACULTY OF ENGINEERING
FINAL EXAMINATION

MATHEMATICS 263

ORDINARY DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

Examiner: Professor S. W. Drury
Associate examiner: Professor G. Schmidt

Date: Tuesday, December 20, 2005
Time: 14:00 – 17:00

Family Name:
Given Names:

Student Number:

Instructions

1. Fill in the above clearly.
2. This is a closed book exam. Faculty standard calculators only are permitted.
3. This examination booklet consists of this cover, Pages 1 through 9 containing questions; Pages 10 and 11, which are blank; and page 12 containing a Laplace transform table.

   You are expected to show all your work. All solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
4. You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)

PLEASE DO NOT WRITE INSIDE THIS BOX

			1 total /10
			2 total /10
			3 total /10
			4 total /10
			5 total /12
			6 total /12
7(i) /2	7(ii) /4	7(iii) /4	7 total /10
8(i) /4	8(ii) /4	8(iii) /4	8 total /12
9(i) /4	9(ii) /6	9(iii) /4	9 total /14
			grand total /100

1. (10 points) Solve the initial value problem

$$\frac{dy}{dx}\cos(x) + 3y\sin(x) = (\cos(x))^2, \quad y(0) = 1.$$

2. (10 points) By making the substitution $u = x - y$, solve the initial value problem

$$\frac{dy}{dx} = (x-y)^2 - 3, \quad y(0) = 0.$$

3. (10 points) By finding an integrating factor of the form $x^p y^q$ solve the implicit initial value problem

$$(5x^2y + 6y^3)dx + (2x^3 + 8xy^2)dy = 0, \quad \text{solution passes through } (x, y) = (1, 1).$$

Leave your answer in implicit form.

4. (10 points) Find the general solution of the equation

$$y''' + 2y'' + y' = 64(x + e^x).$$

5. (12 points) Find the general solution of the equation

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3e^x$$

6. (12 points) Use Laplace transforms to find the solution of the initial value problem

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 13y = 169t + 4\delta_\pi(t), \quad y(0) = 0, \quad \frac{dy}{dt}(0) = 1$$

in $t \geq 0$. Note that a table of elementary Laplace Transforms is provided at the end of this exam. There is no credit for other methods of solution.

7. (10 points) Let $A = \begin{pmatrix} 1 & 1 & a \\ a & a & 1 \\ 1 & -a & -1 \end{pmatrix}$.

(i) (2 points) Find the rank of A for each value of the scalar a.

(ii) (4 points) For each value of a such that A is a singular matrix, find a basis of the kernel of A, i.e. of the solution space for $Ax = 0$.

(iii) (4 points) For each value of a such that A is a singular matrix, find a basis of the column space of A.

8. (12 points) Consider the matrix

$$A = \begin{pmatrix} 9 & 2 \\ 2 & 6 \end{pmatrix}$$

(i) (4 points) Find the eigenvalues of A.
(ii) (4 points) For each eigenvalue, find a corresponding eigenvector.
(iii) (4 points) Find an orthogonal matrix U such that $U'AU$ is diagonal.

9. (14 points) Consider the matrix

$$B = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$

(i) (4 points) Find the characteristic polynomial and the eigenvalues of B.
(ii) (6 points) Compute the matrix $\exp(tB)$ where t is a scalar.
(iii) (4 points) Solve the initial value problem

$$\begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \qquad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

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